# Example 9-1.
## Isomorphous phase diagrams and the lever law.

For a copper-nickel alloy containing 40% Ni, determine the composition of each phase present at 1300C, 1270C, 1250C, and 1200C.

Actually, we will do a little more than that in this example. The problem asks only for the identification and composition of each phase, but we will also consider the amount or proportion of each phase present, and the evolution of the microstructure of the sample during cooling. These are the same kinds of questions considered elsewhere in this Chapter, and in the context of Chapter 10, when the slightly more complicated eutectic phase diagram is introduced.

For an isomorphous diagram like copper-nickel, there is unlimited solid solubility in both the liquid and solid phase. However, the process of freezing (or melting) does not occur at a single fixed temperature, as in a pure component, but over a temperature range. At each temperature between the liquidus and solidus lines, the equilibrium structure is a mixture of the two (solid and liquid) phases, with the amounts and compositions of the phases fixed. These values can be determined from the phase diagram.

There are only two possibilities in any two-component material. A point on the phase diagram may either:

A. lie within a single phase field, in which there are two degrees of freedom, and the temperature and composition can be varied with no change in the microstructure. Single phase fields are usually marked on phase diagrams with either a name or a greek letter.

B. lie within a two-phase field, in which the proportion of the two phases will vary with composition or temperature changes. It is in these regions that the tie-line and the lever law are used. (a common "silly" mistake is to try to draw a tie line in a single phase region).

The basic questions that the phase diagram can be used to answer are:

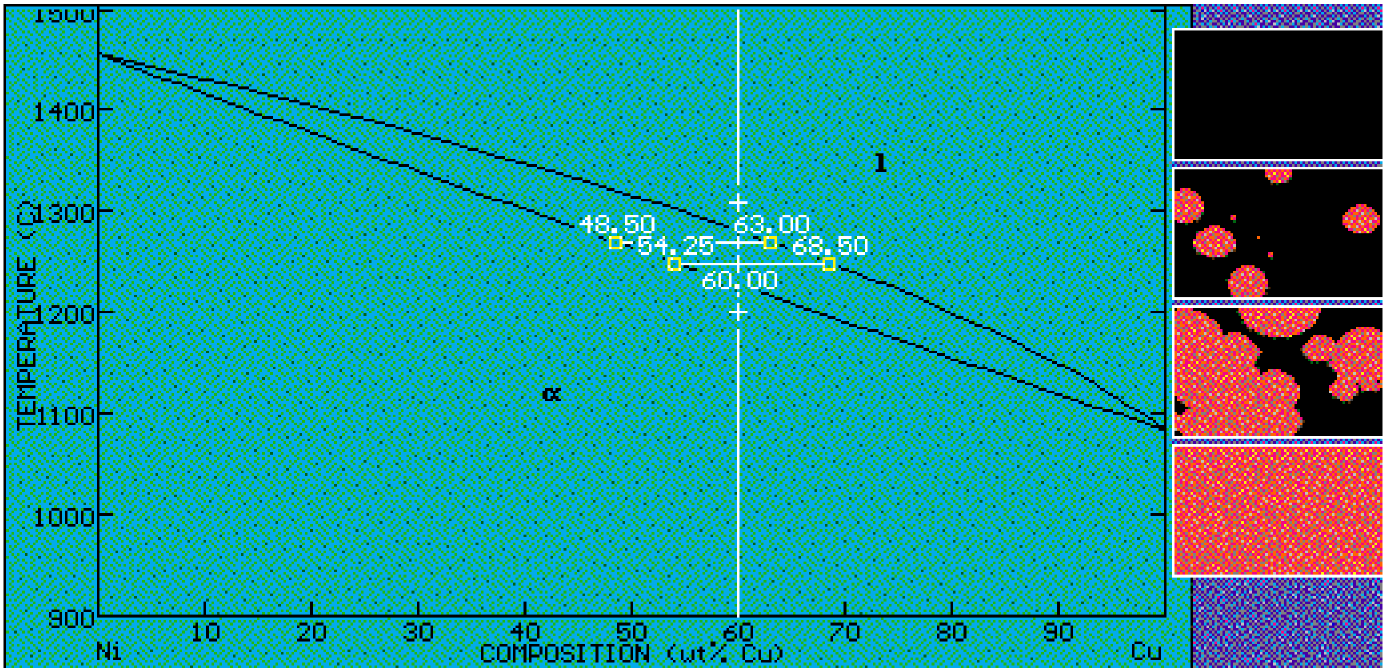
A. What phase or phases are present at a particular overall composition and temperature (in a single phase region, this is just the one phase; in a two-phase field, the ends of the tie line identify the two phases present)?

B. What are the compositions of the phases present (in a single phase region, this is just the bulk composition; in a two phase field, the ends of the tie line give the answer)?

C. What are the amounts or proportions of the phases (in a single phase region, this is a "trick" question since the phase is 100%; in a two-phase field, the lever law is used to get the answer)?

D. On slow (equilibrium) cooling, what would the arrangement of those phases into microstructure look like?

Using the example of the copper-nickel diagram, examples of answers to these questions will be examined below for the case of a composition of 60%Cu - 40%Ni, at temperatures of 1300C, 1270C, 1250C, and 1200C.




What phases are present, and what are their compositions?

1300C. This is a point in a single phase field. Therefore, the only phase present is the liquid, its composition is the same as the overall composition of the sample (60%Cu-40%Ni), and the liquid is 100% of the sample.

1270C and 1250C. This is a point in the two-phase region. The ends of the tie line intersect the phase boundaries of the solid alpha phase (to the left) and the liquid (to the right). These are therefore the two phases present. As marked on the diagram, the points at which the tie lines intersect the phase boundaries can be read from the horizontal concentration scale to indicate the composition of the phases. At each temperature, the compositions of the phases are therefore as listed below. Remember that the axis shows the weight percent of Cu, so the balance must be Ni. Also, note that the diagrams in the book can usually only be read to the nearest 1 or 2%.

| Temperature | Solid phase | Liquid |
|---|---|---|
| 1270 | 48.50% Cu - 51.50% Ni | 63.00% Cu - 37.00% Ni |
| 1250 | 54.25% Cu - 45.75% Ni | 68.50% Cu - 31.50% Ni |

1200C. This is a point in a single phase field. Therefore, the only phase present is the solid, its composition is the same as the overall composition of the sample (60%Cu - 40%Ni), and the solid is 100% of the sample.

What are the amounts or proportions of the phases?

This question only needs to be answered for the 1250C and 1270C points, since it was noted above that the 1200C and 1300C points lie in single phase fields which therefore comprise the entire sample at those temperatures.

The lever law can be used to determine either phase. We use the length of the portion of the tie line AWAY from the phase, divided by the total length of the tie line, to get the fraction of the phase. The lengths are conveniently measured by subtracting the concentration of Cu at each end of the line. Remember that the overall composition of the alloy is 60% Cu - 40% Ni.

$Conc_{overall} = 60$

$Conc_{Liquid} = 68.5$ (this is an example using the data for 1250C from the table above)

$Conc_{Solid} = 54.25$

Now these can be dragged into the following equation to solve for the amount of solid at this temperature. They can be dragged in one-at-a time, or you can use shift-click to select all at once. In the first method, click on each of the given values (easiest way is to click on the equal sign, which selects the whole equation), depress the option key, and then drag it to the corresponding symbol in the equation, which will highlight. Release the mouse to substitute the value. Do this sequentially for each of the three values. In the second method, click on the first expression, then depress the shift key and click on each of the others (they will all highlight). Then depress the option key and drag then to the square marker for the entire equation, which will highlight. Release the mouse and all of the values will be substituted at once.

$$AmountofSolid = \frac{Conc_{Liquid} - Conc_{overall}}{Conc_{Liquid} - Conc_{Solid}}$$

$$AmountofSolid = 0.59649$$

This says that the sample is 59.6% solid, 40.5% liquid at this temperature.

The "drag and drop" interface of Theorist is really overkill for this kind of very simple calculation.It is generally easier to just type in the fraction with the correct numeric values, and then evaluate it directly. This is still preferable to using a pocket calculator, since the structure of the equation is evident and you are less likely to make a silly mistake with parentheses or order of operations.

$$AmountofSolid = \frac{68.5 - 60}{68.5 - 54.25}$$

$$AmountofSolid = 0.59649$$

This gives the identical answer (of course). It says that the fraction of solid phase present at 1250C is 0.596 or 59.6%. In the sketch of the microstructure shown at the right side of the diagram, the amount of red solid phase should therefore be about 60% of the area of the sample. If we repeat the calculation for the 1270C temperature, the fraction of alpha changes as shown below.

$$AmountofSolid = \frac{63 - 60}{63 - 48.5}$$

$$AmountofSolid = 0.2069$$

We could set up the same type of lever law calculation for the amount of liquid, in which case the numerator of the fraction would be the difference in concentration of the overall sample (60.00% Cu) and the concentration of the solid phase. However, it is easier to just subtract the fraction of solid from 1.0, since the part of the sample that is not solid must be liquid (those are the only two phases present).

| Temperature | Amount of Solid | Amount of Liquid |
|---|---|---|
| 1300 | 0% | 100% |
| 1270 | 20.69% | 79.31% |
| 1250 | 59.65% | 40.35% |
| 1200 | 100% | 0% |

These number indicate that as the sample is cooled through the two-phase solid + liquid region, solidification proceeds with the solid (alpha) phase increasing from 0% (just as freezing begins at the liquidus) to 100% of the sample (as the temperature drops to the solidus). This is exactly the trend that the sketches shown with the phase diagram indicate. They show a random pattern of nucleation and growth of solid particles, with the extent of the solid increasing as temperature is lowered.

It is important to understand how these different results are interconnected. The lever law (and the sketches) indicate the amount of each phase. The ends of the tie line give the composition of the phases. As the solidification of the solid phase proceeds on cooling, the liquid is progressively depleted in copper (which goes into the solid phase), and so it increases in nickel content. Since the sample AS A WHOLE does not change composition during this cooling process, the change in the amount and composition of the phases must be interconnected (through the lever law) in order to maintain mass balance for the sample.